Design of Parallel Cascade Control for Disturbance-Rejection

A new approach is presented to overcome load disturbance problems. Based on the parallel cascade control structure, the primary controller is designed for the servo response, and the secondary controller is designed for the disturbance-rejection purpose. A perfect disturbance-rejection controller is derived for the secondary loop. Simulation studies show that improvement in load responses can be achieved for some cases. However, for some other cases, parallel cascade control simply worsens the load responses as a result of interaction between primary and secondary loops. A new interaction measure for load disturbance (γ) is proposed to determine whether improved load responses is achievable using cascade control structure. More importantly, the interaction measure also quantifies the margin of improvement that can be achieved over conventional single-loop control. A design procedure for improved load responses was proposed. The simulation results show that the new approach offers a simple and effective alternative for disturbance-rejection.

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Introduction

Disturbance-rejection is the major concern in the design of process control systems. In the absence of disturbances, a process will remain at the steady state, and control is not necessary. Several control algorithms were proposed to improve the disturbance-rejection capability: dual algorithm (Luyben and Shunta, 1972; Buckley et al., 1985); two-port control (Stephanopoulos and Huang, 1986); generalized analytical predictor (Wellons and Edgar, 1987). Incorporated with process model, the effect of load disturbance can be estimated. Two controllers can be designed separately for servo responses and disturbance-rejection. This type of approach increases the complexity of control system design by designing two controllers for a single pair of input and output.

Using the combination of primary and secondary measurements, a simpler approach can be taken to overcome the load disturbance problems. This is the familiar parallel cascade control structure (Luyben, 1973a), Figure 1A. The control objective is to maintain the primary output, x_p, at the setpoint in the face of disturbances. The output of the primary controller, K_p, resets the setpoint of the secondary loop. The secondary measurement can be selected to infer the load disturbance. A secondary controller, K_s, can be designed for the disturbance-rejection purpose, while the primary controller maintaining good servo responses. Parallel cascade structure seems to offer an effective solution to overcome disturbance problems. However, in most industrial applications, parallel cascade control was designed most often for dynamics (to overcome time delay in the primary measurement) and reliability purposes. For example, temperature/composition cascade control of a distillation column (Luyben, 1973a) uses the accurate and less reliable primary measurement (composition) to reset the set point of less accurate and more reliable secondary measurement (temperature). The faster secondary measurement also compensates the slow dynamics of the primary measurement. In most cases PI controllers were used in both loops (Yu and Luyben, 1986). Despite wide applications of parallel cascade control, little research has been done on this simplest type of multivariable systems. The design of parallel cascade system remains an empirical approach with little theoretical basis.

The purpose of this paper is to explore the disturbance-rejection capability of parallel cascade control systems. A perfect disturbance-rejection controller, K_s, is derived. An interaction measure, similar to relative disturbance gain (RDG) (Stanley et al., 1985), is proposed. This measure can be calculated from steady-state information only. It relates quantitatively to the margin of improvement in disturbance-rejection that can be

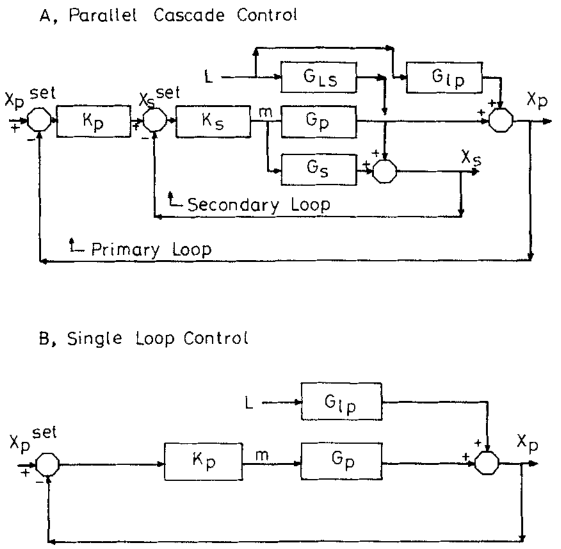

Figure 1. Parallel cascade control (*A*) and single-loop control (*B*).

made by parallel cascade control over the conventional single-loop control. Finally, a design procedure for better disturbance-rejection is proposed. Several cases are studied to illustrate this procedure.

Perfect Disturbance-Rejection

Parallel cascade control structure

Parallel cascade control is the simplest type of multivariable control structure. The open-loop process transfer function matrix, relating the primary and secondary outputs, x_p and x_s, to the manipulated and load variables, m and L, is:

$$\begin{bmatrix} x_p \\ x_s \end{bmatrix} = \begin{bmatrix} G_p & G_{Lp} \\ G_s & G_{Ls} \end{bmatrix} \begin{bmatrix} m \\ L \end{bmatrix} \tag{1}$$

The subscript L denotes load variable. The subscripts p and s denote primary and secondary loop, respectively. Note that this is a nonsquare multivariable systems with two controlled variables and one manipulated variable. In the parallel cascade control structure, the primary output, x_p, is assumed to be much more important than the secondary output, x_s. The control objective is to maintain x_p at the set point. The output of primary controller, K_p resets the setpoint of the secondary controller, K_s, Figure 1A. The closed-loop relationship for the primary output is:

$$x_p = \frac{K_s K_p G_p}{1 + K_s(G_s + G_p K_p)} x_p^{set} + \frac{G_{Lp} + K_s(G_{Lp}G_s - G_{Ls}G_p)}{1 + K_s(G_s + G_p K_p)} L \tag{2}$$

Unlike the closed-loop load transfer function of single-loop control, Figure 1B, the secondary controller, K_s, appears explicitly on the numerator of the closed-loop load transfer function (the second term on the RHS). Therefore, it is possible to design K_s such that improved disturbance-rejection can be achieved.

Designing K_s for disturbance-rejection

Conventionally, controllers with integral action (in most circumstances PI controllers) were used in both loops. To achieve better disturbance-rejection, the closed-loop load transfer function, $G_{Lp,CL}$, should be kept as small as possible (p. 409, Luyben, 1973b).

$$G_{Lp,CL} = \frac{G_{Lp} + K_s(G_{Lp}G_s - G_{Ls}G_p)}{1 + K_s(G_s + G_p K_p)} \tag{3}$$

One way to achieve this is to make the denominator as large as possible. This is equivalent to design a perfect controller. However, this approach is limited by the stability consideration (at ultimate frequency $|K_s + (G_s + G_p K_p)|$ cannot exceed 1). Another approach is to design K_s, a perfect disturbance-rejection controller, such that the numerator can be kept at zero. Let

$$G_{Lp} + K_s(G_{Lp}G_s - G_{Ls}G_p) = 0 \tag{4}$$

Then, the secondary controller, K_s, for perfect disturbance-rejection becomes:

$$K_{s(s)} = \frac{1}{\left[\dfrac{G_{Ls}G_p}{G_{Lp}G_s} - 1\right] G_s} \tag{5}$$

Several observations can be drawn immediately. Firstly, the perfect disturbance-rejection controller is not restricted to a PI controller (as being designed conventionally). It depends on the system parameters of G_p, G_s, G_{Lp} and G_{Ls}. In most cases, the controllers are of the form of proportional or proportional plus lead/lag types. Secondly, perfect disturbance-rejection cannot be achieved for following reasons: 1. K_s is not physically realizable (e.g., K_s contains prediction terms); and 2. K_s results in a positive feedback controller in the secondary loop. It is understandable that a perfect disturbance-rejection controller cannot be realized in practice because of prediction terms (e.g., e^{Ds}). However, it is interesting to see that a perfect disturbance-rejection controller may require a positive feedback loop (e.g., the sign of controller, K_s, differs from the sign of process transfer function, G_s). The dimensionless ratio, $G_{Ls}G_p/G_{Lp}G_s$, plays a crucial role to determine whether perfect disturbance-rejection requires a positive feedback secondary loop.

New Interaction Measure for Load Disturbance

To improve the disturbance-rejection capability of cascade control systems, the interaction between the primary and secondary loops was analyzed.

Definition

An interaction measure for a load disturbance is defined as:

$$\gamma_{(s)} \equiv \frac{(G_{Ls(s)}/G_{s(s)})}{(G_{Lp(s)}/G_{p(s)})} \tag{6}$$

$\gamma_{(s)}$ is a function of frequency. For most chemical processes, the dynamics (e.g., time constants) of the process transfer function, G_p or G_s, is similar to that of load transfer function, G_{Lp} or G_{Ls}.

As the result of cancellation, the interaction measure γ should be fairly constant over a wide frequency range in most cases. Without loss of generality, only the steady-state aspect of $\gamma(\gamma_{(0)})$ is discussed in this paper.

If $\gamma > 1$, the perfect disturbance-rejection is possible at least from steady-state point of view. The larger the value of γ, the smaller the controller gain, $K_{s(0)}$, should be (Eq. 5). For systems with $\gamma < 1$, the perfect disturbance-rejection controller requires a positive feedback secondary loop. This type of positive feedback loop usually is not desirable in practice despite the fact that the whole system may be closed-loop stable. It is interesting to note that integral action is required in the secondary controller to achieve perfect disturbance-rejection for systems with $\gamma = 1$, since K_s approaches infinity as s goes to zero (Eq. 5). The interaction measure γ is dimensionless and independent of input-output scaling. It indicates whether perfect disturbance-rejection can be achieved from steady-state point of view.

Physical meaning

To understand the interaction between primary, x_p, and secondary, x_s, variables, the movement of manipulated variable, m, under different control strategies (controlling x_s or x_p) is analyzed. Physically, γ can be defined as:

$$\gamma \equiv \frac{\left[\frac{\partial m}{\partial L}\right]_{x_s}}{\left[\frac{\partial m}{\partial L}\right]_{x_p}} \tag{7}$$

The numerator denotes movement of the manipulated variable in the face of a load change when x_s is under perfect control. The denominator represents movement of the manipulated variable when x_p is kept constant. The magnitude and direction of the control action play an important role in deciding how a perfect disturbance-rejection controller should be designed. γ is the extention of RDG (relative disturbance gain) (Stanley et al., 1985; Marino-Galarrga et al., 1987a,b) to nonsquare systems. The definition proposed here differs from that of Stanley et al. (1985).

The physical interpretation for a system with $\gamma > 1$ is that, in face of the load disturbance, keeping x_p constant requires less control action than keeping x_s constant, Figure 2A. From control point of view, a positive steady-state offset, $|x_s^{set}| > |x_s|$, in the secondary variable is required. This can be accomplished using a proportional type of controller in a negative feedback loop. In a system with $\gamma = 1$, controlling x_p requires exactly the same amount of manipulation as controlling x_s, Figure 2B. Without modeling error, controlling x_s is essentially the same as controlling x_p in this case. Therefore, a controller with integral action (e.g., a PI controller) is needed for K_s to keep x_p on the set-point. For a system with $1 > \gamma > 0$, controlling x_p requires more control action than controlling x_s, Figure 2C. From x_s point of view, a negative steady-state offset ($|x_s| > |x_s^{set}|$) is needed to keep x_p constant. Unfortunately, this can only be accomplished using an open-loop unstable controller with the overall closed-loop gain, $|G_{s(0)}K_{s(0)}|$, greater than 1 (since the only way to stabilize a positive feedback system with overall closed-loop gain greater than 1 is to design an open-loop unstable controller). For a system with $\gamma < 0$, controlling x_s leads to wrong control action when the load disturbance entering the process. Figure 2D shows that controlling x_p requires the manipulated variable moving toward the direction opposite to that of controlling x_s. From control point of view, this can be accomplished by designing a positive feedback controller with the overall closed-loop gain less than 1 (Eq. 5).

Only qualitative aspect of γ is discussed thus far. To evaluate

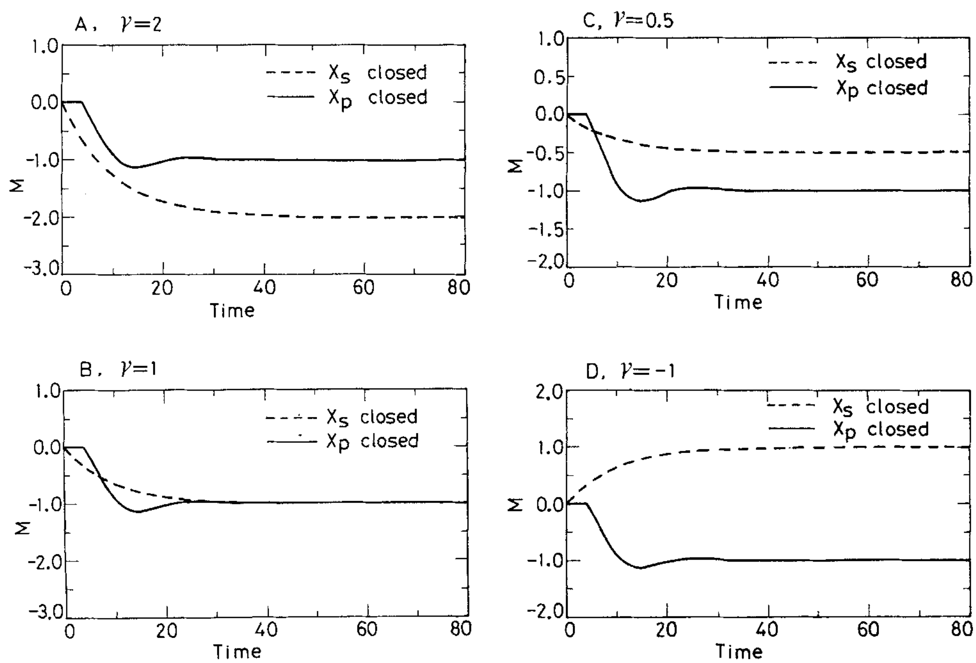

Figure 2. Movement of control action when x_s or x_p is closed for a step load change for systems with: $\gamma = 2$ (*A*), $\gamma = 1$ (*B*), $\gamma = 0.5$ (*C*) and $\gamma = -1$ (*D*).

control system performance, quantitative aspects of γ is analyzed in the next section.

Relationship with integrated error

The interaction measure γ is related quantitatively to the integrated error (IE), area under response curve (Stanley et al., 1985). The integrated error for the primary output, x_p, is defined as:

$$\mathrm{IE} \equiv \int_0^\infty (x_p^{\mathrm{set}} - x_{p(t)})\, dt = \int_0^\infty e_{p(t)}\, dt \tag{8}$$

In terms of Laplace transformed variable, IE can be expressed as (Stanley et al., 1985):

$$\mathrm{IE} = \lim_{s \to 0} E_{p(s)} \tag{9}$$

where $E_{p(s)}$ is the Laplace transformation of $e_{p(t)}$ in Eq. 8. It is important to note that Eq. 9 holds for closed-loop stable systems with zero steady-state offset.

To test the effectiveness of parallel cascade control systems, comparison was made between systems with and without cascade control structure (a system without cascade control structure is simply the conventional single-loop control, Figure 1B). The logical basis for comparison is to make the IE's be the same for both structures with a step set-point change.

$$\mathrm{IE}_{PC} = \mathrm{IE}_{SL} \tag{10}$$

Subscripts PC and SL stand for parallel cascade control and single-loop control respectively. This required that the controller parameters follow the relationship:

$$\frac{(K_{c,p})_{PC}(\tau_{I,p})_{SL}}{(K_{c,p})_{SL}(\tau_{I,p})_{PC}} = \frac{1 + G_{s(0)}K_{s(0)}}{K_{s(0)}} \tag{11}$$

where $K_{c,p}$ and $\tau_{I,p}$ are controller gain and reset time for the primary loop.

Since the disturbance-rejection capability is our major concern, the IE's for a unit step change in the load variable L are:

$$\mathrm{IE}_{SL} = \frac{G_{Lp(0)}}{\dfrac{(K_{c,p})_{SL}}{(\tau_{I,p})_{SL}} G_{p(0)}} \tag{12}$$

$$\mathrm{IE}_{PC} = \frac{G_{Lp(0)} + K_{s(0)}(G_{Lp(0)}G_{s(0)} - G_{Ls}(0)\,G_{p(0)})}{\dfrac{(K_{c,p})_{PC}}{(\tau_{I,p})_{PC}} K_{s(0)}G_{p(0)}} \tag{13}$$

Substituting Eq. 11 into Eq. 12 and dividing Eq. 13 by Eq. 12 gives:

$$\frac{\mathrm{IE}_{PC}}{\mathrm{IE}_{SL}} = 1 - \frac{G_{s(0)}K_{s(0)}}{1 + G_{s(0)}K_{s(0)}}\gamma \tag{14}$$

Equation 14 is one of the most important results of this paper. It indicates that the margin of improvement in disturbance-rejection can be achieved by parallel cascade control over conventional single-loop control. For $\gamma > 1$, $K_{s(s)}$ can be designed such that improved disturbance-rejection can be achieved. If K_s was designed according to Eq. 5, the integrated error, IE_{PC} becomes zero. The premultiplier before γ in Eq. 14 is positive under most circumstances. Therefore, for systems with $\gamma < 0$, parallel cascade control structure simply worsens the disturbance-rejection capability based on the IE analyses. For systems with $1 > \gamma > 0$, better load responses can be obtained with cascade structure (Eq. 14). However, improved IE_{PC} can be achieved by using integral action in the secondary controller (the premultiplier before γ is 1 in this case). For the secondary loop also under integral control, Eq. 14 becomes:

$$\frac{\mathrm{IE}_{PC}}{\mathrm{IE}_{SL}} = 1 - \gamma \tag{15}$$

It is interesting to note that the conventional approach (use PI controllers in both loops) works well for systems with $1 \geq \gamma > 0$.

In summary, for $\gamma > 0$, better disturbance-rejection can be achieved with parallel cascade structure. For $\gamma < 0$, parallel cascade structure worsens load responses. In this paper, IE is used for quantitative analysis instead of IAE or ISE, since only steady-state information is required. Simplicity of IE analysis has its value in the environment requiring high efficiency with incomplete knowledge. Moreover, in most cases, the primary controlled variable in parallel cascade control is product composition. Over a long period of time, IE is more appropriate to represent true product composition than IAE or ISE.

Design Procedure

Based on the ongoing analyses, a simple design procedure for better disturbance-rejection is proposed. The basic principle is to design the primary controller, K_p, for servo response and the secondary controller, K_s, for disturbance-rejection. Three cases were studied.

Case of $\gamma > 1$

Since steady-state version of perfect disturbance-rejection controller is proposed for this case, stability with respect to magnitude of γ and noninvertible element (in $G_{s(s)}$) is analyzed. First, design the secondary controller, K_s, using Eq. 5. In most cases, K_s is P-only or proportional-plus-lead/lag type of controller. For simplicity, P-only controller is suggested.

$$K_s = \frac{1}{(\gamma - 1)G_{s(0)}} \tag{16}$$

The closed-loop characteristic equation (CLCE) for the secondary loop becomes:

$$1 + G_sK_s = 1 + \frac{1}{\gamma - 1}G_s' \tag{17}$$

where G_s' is the normalized secondary transfer function, $G_{s(0)}' = 1$. It can be seen that the magnitude of γ has very strong effect on the stability. For systems with $\gamma > 2$, stability is guaranteed for G_s with monotonic step response (can be proved using Bode small gain theorem). Therefore, noninvertible element, e^{-Ds}, in G_s does not pose any stability problem for $\gamma > 2$. For G_s with nonmonotonic step response (including RHP zeros and underdamped systems), stability of the closed-loop system should be checked for small value of γ (γ around and less than 2). Problems may arise for systems with the value of γ very close to 1. For example, the CLCE for a system with $\gamma = 1.001$ is:

$$1 + 1{,}000 \cdot G_s' = 0$$

Very likely, the secondary loop cannot be stabilized by itself. If stability of the secondary loop is violated, controller with integral action is recommended.

Next, design the primary controller, K_p, to meet servo response specification. The integrated error for servo response

$$\mathrm{IE}_{PC} = \frac{1 + K_{s(0)}G_{s(0)}}{\frac{(K_{c,p})_{PC}}{(\tau_{I,p})_{PC}} K_{s(0)}G_{p(0)}} \tag{18}$$

can be used as a quick measure of performance.

Case of $1 \geq \gamma > 0$

Controllers with integral action are suggested for this case. First, design the primary PI controller, K_p, using any suitable tuning methods such as IMC tuning (Rivera et al., 1986), Buckley's tuning method (p. 420, Luyben, 1973b), and Ziegler-Nichols tuning. The reason for designing the outside loop first is that the IE_{PC} for setpoint response depends solely on the controller parameters of primary loop. The controller parameters for a parallel cascade and single-loop control with the same IE for the servo response can be expressed as:

$$\frac{(K_{c,p})_{PC}}{(\tau_{I,p})_{PC}} = \frac{(K_{c,p})_{SL}}{(\tau_{I,p})_{SL}} G_{s(0)} \tag{19}$$

Second, design the secondary controller to meet desired stability margin (Eq. 2 for the closed-loop characteristic equation).

It should be emphasized that, despite the fact that the IE_{PC} is independent of controller parameters (Eq. 15), it does not mean IAE or ISE is independent of controller parameters. If the absolute value of x_p is of major concern (e.g., x_p is the temperature in a reactor), a careful selection of tuning constants for K_s is necessary. In this paper, IMC tuning method is used throughout.

Case of $\gamma < 0$

Cascade control is not recommended in this case (Eqs. 14 and 15). The load responses are much poorer for cascade control than conventional single-loop control regardless of how sophisticated the controllers are.

Design procedure is proposed for three distinct regions, but the boundary is obviously fuzzy (especially for $0 < \gamma < 2$). Parallel cascade control is not recommended for the combination of x_p and x_s results in $\gamma < 0$. It does not mean parallel cascade control is not recommended for the process unit. γ can be served as a criterion for the selection of secondary measurement.

Applications

Four different cases for values of γ ranging from 2 to -1 were studied. Table 1 gives system parameter values. Controller parameter values were obtained from IMC tuning method.

The performance of cascade control was compared with single-loop control. Each system gives the same servo response according to IE's, Figure 3. However, significant different responses were observed for a unit step load change, Figure 4. For $\gamma \geq 0$, improved disturbance-rejection can be achieved using parallel cascade control structure, Figure 4. For $\gamma < 0$, parallel cascade just worsens the load response, Figure 4.

Table 1. Process and Controller Parameters for the System Studied

Primary Loop	Seondary Loop
$G_p = \frac{e^{-4s}}{20s+1}$	$G_s = \frac{1}{10s+1}$
$G_{Lp} = \frac{e^{-4s}}{20s+1}$	$G_{Ls} = \frac{G_{Ls(0)}}{10s+1}$

Case No.	γ	$G_{Ls(0)}$	$(K_{c,p})_{PC}$*	$(\tau_{I,p})_{PC}$*	$(K_{c,s})_{PC}$*	$(\tau_{I,s})_{PC}$*
1	2	2	4.58	15.6	1	—
2**	1	1	3.24	22.0	1	10
3	0.5	0.5	3.24	22.0	1	10
4	−1	−1	3.24	22.0	1	10

*The controller parameters were obtained using IMC tuning method (cases 2, 3 and 4) or modified IMC tuning method.
**Controller parameters for the single-loop control is the same as case 2.

Conclusions

The concept of parallel cascade control can be applied to improve disturbance-rejection capability. However, improvement can be made only on certain systems. A new interaction measure for load disturbance, γ, is proposed to determine whether better disturbance-rejection can be achieved. γ is related closely to the concept of RDG. Therefore, the physical interpretation of γ becomes obvious. More importantly, γ is a measure of the margin of improvement in integrated errors that can be achieved using parallel cascade control structure over conventional single-loop control (Eqs. 14 and 15). This gives process control engineers a quantitative measure of control loop performance based on steady-state information only. A design procedure was proposed for the design of cascade control systems with improved disturbance-rejection capability.

For the control engineer who tries to achieve better disturbance-rejection, the dual algorithm of Shunta and Luyben (1972) and two-port control of Stephanopoulos and Huang (1986) offer two approaches. The parallel cascade control structure offers an effective and realistic alternative.

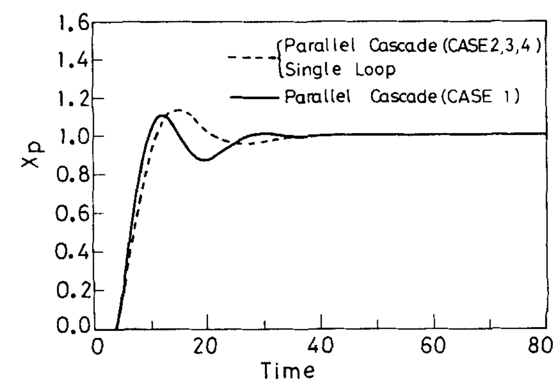

Figure 3. Servo responses for single-loop control and parallel cascade control (cases 1, 2, 3 and 4).

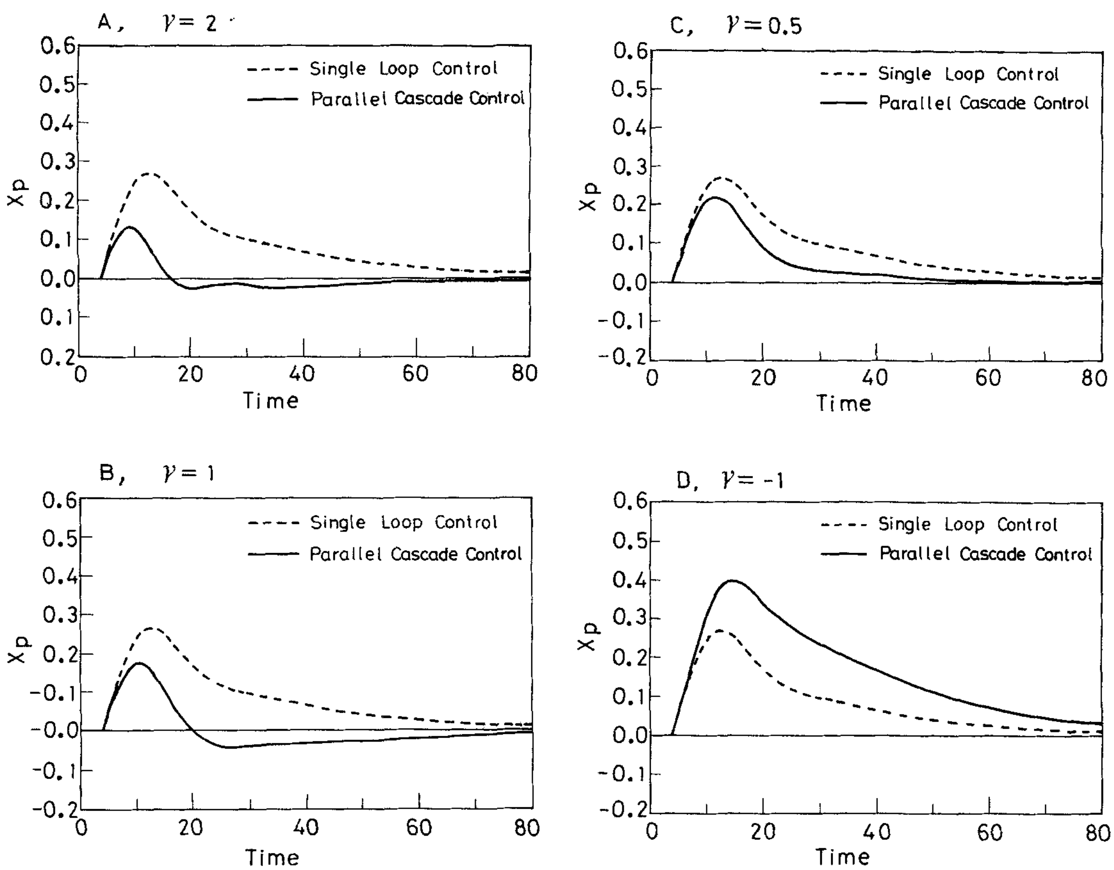

Figure 4. Load responses for single-loop control and parallel cascade control (*A*, case 1; *B*, case 2; *C*, case 3; *D*, case 4).

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Notations

e = error between set point and controlled variable as a function of time
E = Laplace transformation of e
IAE = integrated absolute error
IE = integrated error
ISE = integrated square error
G = process transfer function
G' = normalized process transfer function ($G'_{(0)} = 1$)
G_L = load transfer function
K = controller transfer function
K_c = controller gain
L = load variable
m = manipulated variable
RDG = relative disturbance gain
RHP = right half plane
x = controlled variable

Greek letters

γ = *interaction* for load disturbance
τ_I = reset time

Superscript

set = setpoint

Subscript

CL = closed loop
p = primary loop
PC = parallel cascade control
s = secondary loop
SL = single-loop control

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